

SINGLE CORRECT

- If $a, b, c, d \in R$ such that $\frac{a+2c}{b+3d} + \frac{4}{3} = 0$, then the equation $ax^3 + bx^2 + cx + d = 0$ has

(a) atleast one root in $(-1, 0)$ (b) atleast one root in $(0, 1)$
(c) no root in $(-1, 1)$ (d) no root in $(0, 2)$.
- If $A = \{1, 2, 3, 4\}$, $B = \{1, 2, 3, 4, 5, 6\}$ and $f: A \rightarrow B$ is an injective mapping satisfying $f(i) \neq i$, then number of such mapping are

(a) 182 (b) 181 (c) 183 (d) None of these
- Sum of first 10 terms of the series, $S = \frac{7}{2^2 \cdot 5^2} + \frac{13}{5^2 \cdot 8^2} + \frac{19}{8^2 \cdot 11^2} + \dots$ is

(a) $\frac{255}{1024}$ (b) $\frac{88}{1024}$ (c) $\frac{264}{1024}$ (d) $\frac{85}{1024}$
- Let f be integral over $[0, a]$ for any real positive a . If $I_1 = \int_0^{\frac{\pi}{2}} \cos \theta f(\sin \theta + \cos^2 \theta) d\theta$ and $I_2 = \int_0^{\frac{\pi}{2}} \sin 2\theta f(\sin \theta + \cos^2 \theta) d\theta$, then the value of $\frac{I_1}{I_2}$ is

(a) 1 (b) -1 (c) 2 (d) -2
- Let 'f' is a function, continuous on $[0, 1]$ such that $f(x) \leq \sqrt{5} \forall x \in [0, 1]$ and $f(x) \leq \frac{2}{x} \forall x \in \left[\frac{1}{2}, 1\right]$, then the smallest 'a' for which $\int_0^1 f(x) dx \leq a$ holds for all 'f' is

(a) $\sqrt{5}$ (b) $\frac{\sqrt{5}}{2} + 2 \ln 2$ (c) $2 + \ln\left(\frac{\sqrt{5}}{2}\right)$ (d) $2 + 2 \ln\left(\frac{\sqrt{5}}{2}\right)$
- If every root of the polynomial $f(x) = 0$ is real and distinct, then the equation $f''(x)f(x) - [f'(x)]^2 = 0$ has

(a) only imaginary roots (b) only real roots
(c) real and distinct roots (d) real and equal roots
- If the sides a, b are constant and the base angle A, B vary in a triangle ABC , then $\frac{dA}{dB}$ is equal to

(a) $\sqrt{\frac{a^2 - b^2 \sin^2 A}{b^2 + a^2 \sin^2 B}}$ (b) $\sqrt{\frac{a^2 + b^2 \sin^2 A}{b^2 + a^2 \sin^2 B}}$ (c) $\sqrt{\frac{a^2 + b^2 \sin^2 A}{b^2 - a^2 \sin^2 B}}$ (d) none of these
- The value of the integral $\int_0^{\infty} f\left(x + \frac{1}{x}\right) \cdot \frac{\ell n x}{x} dx$, is equal to

(a) $1/2$ (b) 0 (c) 2 (d) e
- The value of $\lim_{n \rightarrow \infty} n \int_0^{\pi/2} \left(1 - \sqrt[n]{\sin x}\right) dx$ is equal to

(a) $\frac{\pi}{2} \ell n 2$ (b) $\pi \ell n 2$ (c) $-\pi \ell n 2$ (d) none of these

10. The inequality $\int_0^1 \frac{dx}{1+x^n} > 1 - \frac{1}{n}$, where $n \in N$, is true for
 (a) $n = 1$ only (b) $n = 2$ only (c) $n \geq 2$ only (d) none of these
11. If $f(x) = \int_0^x \frac{dt}{[f(t)]^2}$ and $\int_0^{2/3} \frac{dt}{[f(t)]^2} = \sqrt[3]{2}$, then the value of $f(72)$ is
 (a) 3 (b) 2 (c) 6 (d) none of these
12. The value of $\int \frac{e^x(x^3 - 3x^2 + 5x - 1)}{(x^2 + 1)^2} dx$ is equal to
 (a) $e^x \left(\frac{x+2}{x^2+1} \right) + C$ (b) $e^x \left(\frac{x-2}{x^2+1} \right) + C$ (c) $e^x \left(\frac{2x-1}{x^2+1} \right) + C$ (d) $e^x \left(\frac{2x+1}{x^2+1} \right) + C$
13. The least value of $\int_1^x \left| \frac{\sin nx}{x} \right| dx$, where $n \in N$ is equal to
 (a) $\frac{2}{\pi} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \right)$ (b) $\frac{2}{\pi} \left(\frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \right)$
 (c) $\frac{1}{\pi} \left(\frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \right)$ (d) $n \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \right)$
14. The value of $\int \frac{(1+x)\sin x}{(x^2 + 2x)\cos^2 x - (1+x)\sin 2x} dx$ is equal to
 (a) $\frac{1}{2} \ln \left| \frac{\sin x + (1+x)\cos x - 1}{\sin x - (1+x)\cos x + 1} \right|$ (b) $\frac{1}{2} \ln \left| \frac{\sin x + (1-x)\cos x - 1}{\sin x - (1+x)\cos x + 1} \right|$
 (c) $\frac{1}{2} \ln \left| \frac{\sin x - (1+x)\cos x - 1}{\sin x - (1+x)\cos x + 1} \right|$ (d) none of these
15. If $f(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$ is divisible by x^3 and α, β, γ are the distinct roots of the equation $2(x-1)^3 + x^2 - 2x + 1 = 2x^3 + bx + c$, then the value of $(a + 2b + 3c + 4d + 5e + 6f)$ equals
 (a) -2 (b) 0 (c) 2 (d) 17
16. Let $f: R - \left\{ \frac{3}{2} \right\} \rightarrow R, f(x) = \frac{3x+5}{2x-3}$. Let $f_1(x) = f(x), f_n(x) = f(f_{n-1}(x))$ for $n \geq 2, n \in N$, then $f_{2016}(x) + f_{2017}(x)$ is equal to
 (a) $\frac{2x^2+5}{2x-3}$ (b) $\frac{x^2+5}{2x-3}$ (c) $\frac{2x^2-5}{2x-3}$ (d) $\frac{x^2-5}{2x-3}$
17. Let $f(x) = -x^2 + 5x + 6$, If $g(x) = |f(|x| + k) + \lambda|$ is derivable for all $x \in R$ then greatest value of $(\lambda + k)$ is equal to
 (a) $-\frac{39}{4}$ (b) $-\frac{49}{4}$ (c) $\frac{49}{4}$ (d) $\frac{39}{4}$
18. If the equation $x^2 - p|x-1| - 2x + 2p^2 - 7 = 0$ is satisfied by exactly three distinct real values of x , then sum of all possible values of p is
 (a) 0 (b) 2 (c) 4 (d) 8

19. If $f(x) = \frac{(x-1)^3 + (e^{x-1} - 1)\left(\tan^{-1}x - \frac{\pi}{4}\right)(x^3 - x^2 - x + 1)}{\tan^{-1}x}$, then the value of $(f'(1) + f''(1) + f'''(1))$ is equal to
- (a) $\frac{8}{\pi}$ (b) $\frac{12}{\pi}$ (c) $\frac{24}{\pi}$ (d) $\frac{32}{\pi}$

20. If $f(t) = \lim_{x \rightarrow 0} \frac{e^{\frac{k}{6}} \cdot \sin\left(kt \left(\frac{\sin x}{x}\right)^{\frac{k}{x^2}}\right)}{(\sqrt{1+t} - 1)}$, $k \in \mathbb{N}$ then value of $\lim_{x \rightarrow 0} \left(\sum_{k=1}^4 f(t)\right)$ is equal to
- (a) 10 (b) 20 (c) 9 (d) 18

21. $f(x)$ is polynomial function of degree 6, which satisfies $\frac{1}{\sqrt{3}} = \frac{\pi}{6}$ and has local maximum at $x = 1$ and local minimum at $x = 0$ and $x = 2$.

Column-I		Column-II	
(A)	The coefficient of x^6 is	(P)	$-2/3$
(B)	The coefficient of x^5 is	(Q)	0
(C)	The coefficient of x^4 is	(R)	2
(D)	The coefficient of x^3 is	(S)	$-12/5$
		(T)	$2/3$
		(U)	$12/5$

The correct option is

- (a) $P \rightarrow 5; Q \rightarrow 4; R \rightarrow 3; S \rightarrow 2$ (b) $P \rightarrow 5; Q \rightarrow 4; R \rightarrow 3; S \rightarrow 2$
(c) $P \rightarrow 5; Q \rightarrow 4; R \rightarrow 3; S \rightarrow 2$ (d) $P \rightarrow 5; Q \rightarrow 4; R \rightarrow 3; S \rightarrow 2$
22. Let $f(x) = \begin{cases} |1-x^2|, & 0 \leq x \leq 2 \\ \{ \alpha x \} + \beta, & 2 < x < p \end{cases}$ where α, β and $p \in \mathbb{R}$.
- If $f(x)$ is non-derivable at exactly one point in $[0, p)$ then maximum value of $(\alpha + \beta + 4p)$ is equal to
[Note: $\{k\}$ denotes the fractional part function of k .]
(a) 8 (b) 9 (c) 12 (d) 16
23. If area enclosed between the curves $y = \ln(x + e)$ and $x = \ln\left(\frac{1}{y}\right)$ and the axis of x is λ sq. unit, then the value of λ must be :
(a) 5 (b) 7 (c) 1 (d) 2
24. $\int_0^1 \frac{1}{(x^2 + 3x + 2)^2} dx =$
(a) $\frac{2}{3} + 2 \log \frac{4}{3}$ (b) $\frac{4}{3} + \log \frac{2}{3}$ (c) $\frac{1}{3} + \log \frac{3}{2}$ (d) $\frac{2}{3} - 2 \log \frac{4}{3}$
25. Let $g(x)$ be a polynomial of degree one and $f(x)$ be defined by $f(x) = \begin{cases} g(x) & x \leq 0 \\ |x|^{\sin x} & x > 0 \end{cases}$.
- If $f(x)$ is continuous satisfying $f'(1) = f(-1)$, then find $g(-101)$
(a) $101 \sin 1 - 100$ (b) $101 \sin 1 + 100$ (c) $100 \sin 1 - 101$ (d) $100 \sin 1 + 101$.

26. Let f, g be two positive valued functions defined on $[-1, 1]$ such that $f(-x)f(x) = 1$ and g is an even function with $\int_{-1}^1 g(x) dx = 1$. Let $I = \int_{-1}^1 f(x)g(x) dx$. Then which of the following statements is necessarily true.
- (a) $I \geq 1$ (b) $I < 1$ (c) $\frac{1}{3} < I < 3$ (d) $I < 0$
27. If $I_1 = \int_1^2 2^{x^2} dx$, $I_2 = \int_1^2 2^{x^2} (\ln x) dx$, $I_3 = \int_1^2 2^{x^2} (\ln x)^2 dx$, $I_4 = \int_1^2 2^{x^2} (\ln x)^3 dx$ then
- (a) $I_4 > I_3 > I_2 > I_1$ (b) $I_3 > I_4 > I_1 > I_2$ (c) $I_4 > I_2 > I_1 > I_3$ (d) $I_1 > I_2 > I_3 > I_4$
28. The integral value of k if $\int_0^{8\pi} e^{\cos \theta} \cos(\sin \theta) d\theta = k\pi$, is :
- (a) 52 (b) 70 (c) 12 (d) 8
29. Consider a function $f(n)$ defined for all $n \in \mathbb{N}$. The function satisfies the following two conditions:
- (i) $f(1) + f(2) + f(3) + \dots$ to $\infty = 1$
(ii) $f(n) = \{(1-p)p^{-1}\} \{f(n+1) + f(n+2) + \dots$ to $\infty\}$, where $0 < p < 1$. Then $f(2)$ is equal to
- (a) $p(1-p)$ (b) $1-p$ (c) $1+p$ (d) None of these
30. If $f: \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is a polynomial function satisfying the functional equation $f(f(x)) = 6x - f(x)$, then $f(17)$ is equal to ?
- (a) 17 (b) -51 (c) 34 (d) -34.
31. If $\int \frac{dx}{\cos^3 x - \sin^3 x} = A \tan^{-1}(\sin x + \cos x) + B \ln \left| \frac{\cos x + \sin x - \sqrt{2}}{\cos x + \sin x + \sqrt{2}} \right| + C$, then (A, B)
- (a) $\left(-\frac{2}{3}, \frac{-1}{3\sqrt{2}}\right)$ (b) $\left(\frac{2}{3}, \frac{-1}{3\sqrt{2}}\right)$ (c) $\left(\frac{1}{3}, \frac{-1}{\sqrt{2}}\right)$ (d) $\left(\frac{2}{3}, \frac{1}{3\sqrt{2}}\right)$
32. Suppose $x_1 = \tan^{-1} 2 > x_2 > x_3 > \dots$ are positive real numbers satisfying $\sin(x_{n+1} - x_n) + 2^{-(n+1)} \sin x_n \sin x_{n+1} = 0$ for $n \geq 1$. Then the value of the $\lim_{n \rightarrow \infty} x_n$
- (a) $\pi/2$ (b) $3\pi/4$ (c) $\pi/4$ (d) $1/2$
33. If $\alpha, \beta, \gamma, \delta$ be real numbers such that $\alpha^2 + \beta^2 = 9$, $\gamma^2 + \delta^2 = 4$ and $\alpha\delta - \beta\gamma = 6$, then the greatest value of $\alpha\gamma$ is equal to
- (a) 9 (b) 6 (c) 4 (d) 3.
34. Let $\theta(x)$ denote an angle measured in radians which is subtended by a fixed closed interval $[1, 3]$ on the y-axis at a point x on the positive x-axis. If θ_0 is the maximum value of $\theta(x)$. then π/θ_0 is equal to
- (a) 9 (b) 6 (c) 4 (d) 3.
35. If $y = e^{-x} \cos x$ and $y_n + k_n y = 0$, where $y_n = \frac{d^n y}{dx^n}$ and $k_n, n \in \mathbb{N}$ are constants. Then, match the elements of Column I with the elements of Column II

Column-I		Column-II	
(A)	The value of k_2 is	(P)	4
(B)	The value of k_{16} is	(Q)	-256
(C)	The value of k_8 is	(R)	64
(D)	The value of k_4 is	(S)	-18

The correct option is

- (a) P→3; Q→4; R→2; S→1 (b) P→3; Q→2; R→4; S→4
(c) P→2; Q→3; R→1; S→4 (d) P→2; Q→1; R→3; S→4

36. $\frac{1}{2\pi} \left(\frac{\pi^3}{1!3} - \frac{\pi^5}{3!5} + \frac{\pi^7}{5!7} - \dots - \frac{(-1)^{n-1} \pi^{2n+1}}{(2n-1)!(2n+1)} \right)$ is equal to
 (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) 1 (d) None of these
37. For every pair of continuous function $f, g: [0, 1] \rightarrow R$ such that $\max \{f(x); x \in [0, 1]\} = \max \{g(x); x \in [0, 1]\}$. Then which of the following statements is(are) true:
 (a) $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c) \exists c \in [0, 1]$
 (b) $(f(c))^2 + f(c) = (g(c))^2 + 3g(c) \exists c \in [0, 1]$
 (c) $(f(c))^2 + 3f(c) = (g(c))^2 + g(c) \exists c \in [0, 1]$
 (d) Both (a) & (b)
38. If $f(x), g(x)$ are thrice differentiable function on $[0, 3]$ satisfying $f'''(x) = g'''(x), f''\left(\frac{1}{2}\right) = g''\left(\frac{1}{2}\right) = 0, f'(1) = 4, g'(1) = 2$ and $f(3) = 12, g(3) = 3$, then which of the following is(are) TRUE?
 (a) $f(2) - g(2) = 7$ (b) $|f(x) - g(x)| < 5 \Rightarrow x \in]-4, 1[$
 (c) $|f(x) - g(x)| > 5 \Rightarrow x \in]-\infty, -4[\cup]1, \infty[$ (d) All of these
39. Find the value of $\lim_{n \rightarrow 0} n^2 \int_{-1/n}^{1/n} (2006 \sin x + 2007 \cos x) |x| dx$
 (a) 2007 (b) 2006 (c) 1947 (d) None of these
40. Let $L = \lim_{x \rightarrow +\infty} \frac{d}{dx} \int_{2 \sin 1/x}^{\sqrt[3]{x}} \frac{3t^4 + 1}{(t-3)(t^2+3)} dt$. Find the value of $10L$.
 (a) 34 (b) 67 (c) 65 (d) None of these
41. Let $\int_{-\infty}^{\infty} f(x) dx = 1$ and $I = \int_{-\infty}^{\infty} f\left(x - \frac{1}{x}\right) dx$. Then value of $(91)^3$ is :
 (a) perfect square (b) prime number (c) perfect number (d) None of these
42. Let $f(x)$ be a continuous function given by $f(x) = \begin{cases} 2x; -1 \leq x \leq 1 \\ x^2 + ax + b; x < -1 \text{ or } x > 1. \end{cases}$
 Let A be the area of the region in the third quadrant bounded by the curves $x = -2y^2$ and $y = f(x)$ lying on the left of the straight line $8x + 1 = 0$, then the sum of the digits of the value of $(192A + 4)$ is
 (a) 12 (b) 15 (c) 18 (d) None of these
43. Let $f: R \rightarrow R$ be a continuous function such that $f(x) = 3x^2 + \int_0^x e^{-t} f(x-t) dt$. Then, which of the following statement is(are) TRUE?
 (a) The turning point of the curve $y = f(x)$ is (2, 4)
 (b) The area bounded by $y = f(x)$, then X-axis and the ordinates $x = 0$ and $x = 1$ is $\frac{5}{4}$ sq units
 (c) The turning point of the curve $y = f(x)$ is (2, 6)
 (d) The area bounded by $y = f(x)$, the X-axis and the ordinates $x = 0$ and $x = 1$ is $\frac{3}{4}$ sq units

44. Which of the following is/are correct?

(a) The value of $\lim_{x \rightarrow 1} \frac{\int_1^x (t-1) \sin \pi t \, dt}{(x-1)^3} = \frac{\pi}{3}$

(b) The distance of the point of intersection of the planes $\vec{r} \cdot (2\hat{i} + \hat{j}) = 0$, $\vec{r} \cdot (4\hat{j} + 3\hat{k}) = 24$ from the origin is 36

(c) Let $g(x)$ be differentiable on $\mathbb{R}$ and $\int_{\sin t}^1 x^2 g(x) dx = (1 - \sin t)$, where $t \in \left(0, \frac{\pi}{2}\right)$. The value of $g\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2}$

(d) If $x, y \in \mathbb{R}^+$ satisfying $x + y = 3$, then the maximum value of $x^2 y$ is 8

45. A curve is represented parametrically by the equations $x = e^t \cos t$ and $y = e^t \sin t$ where t is a parameter then the relation between the parameter t and the angle α between the tangent to the given curve and the x -axis is given by 't' equals

(a) $\frac{\pi}{2} - \alpha$ (b) $\frac{\pi}{4} + \alpha$ (c) $\alpha - \frac{\pi}{4}$ (d) $\frac{\pi}{4} - \alpha$

46. Consider a quadratic function $f(x) = ax^2 + bx + c$ ($a, b, c \in \mathbb{R}$, $a \neq 0$) and satisfying the following conditions

(i) $f(x-4) = f(2-x) \forall x \in \mathbb{R}$ and $f(x) \geq x \forall x \in \mathbb{R}$

(ii) $f(x) \leq \left(\frac{x+1}{2}\right)^2 \forall x \in (0, 2)$

(iii) The minimum value of $f(x)$ is zero

Then value of the leading coefficient to the quadratic polynomial is

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 1

47. Solution of the differential equation $y' = \frac{3yx^2}{x^3 + 2y^4}$ is

(a) $x^3 y^{-1} = \frac{2}{3} y^3 + c$ (b) $x^2 y^{-1} = \frac{2}{3} y^3 + c$ (c) $xy^{-1} = \frac{2}{3} y^3 + c$ (d) none of these

48. Let $f(x) = \begin{vmatrix} [x] & \{x\} & 1 \\ \{x\} & 1 & [x] \\ 1 & [x] & \{x\} \end{vmatrix}$, where $[x]$ = greatest integer function and $\{x\}$ = fractional part of x .

Then which of the following is incorrect

- (a) $f(x) \geq 0$ for $x \in (-\infty, -1]$
 (b) $f(x)$ has no real root for $x \in (0, 1)$
 (c) for $x \in [0, 1)$, range of $f(x)$ is $(-2, -1]$
 (d) for $x \in [1, 2)$, $f(x)$ has exactly one local minima

49. Solution of $y dx - x dy + (1 + x^2) dx + x^2 \sin y dy = 0$

(a) $\frac{y}{x} + \frac{1}{x} - x + \cos y + c = 0$ (b) $\frac{y}{x} + \frac{1}{x^2} - x + \cos y + c = 0$
 (c) $\frac{2y}{x} - \frac{1}{x^2} - x + \cos y + c = 0$ (d) none of these

50. The value of $\int \sqrt{\frac{(1 - \sin x)(2 - \sin x)}{(1 + \sin x)(2 + \sin x)}} dx$ is

(a) $\ln \left(\frac{t - \sqrt{3}}{t + \sqrt{3}} \right) + 2 \tan^{-1} t + c$ (b) $\ln \left(\frac{t + \sqrt{3}}{t - \sqrt{3}} \right) + \tan^{-1} t + c$
 (c) $\sqrt{3} \ln \left(\frac{t + \sqrt{3}}{t - \sqrt{3}} \right) + \tan^{-1} \frac{2t}{1 - t^2} + c$ (d) $\sqrt{3} \ln \left(\frac{t - \sqrt{3}}{t + \sqrt{3}} \right) + 2 \tan^{-1} t + c$ (where $t = \sqrt{\frac{2 - \sin x}{2 + \sin x}}$)

MULTIPLE CORRECT

51. The graph of the function $y = f(x)$ passing through the point $(0, 1)$ and satisfying the differential equation $\tan^2 x \frac{dy}{dx} = \sec x(1 - y) - \frac{dy}{dx}$. Then which of the following statements is(are) TRUE
 (a) It is continuous function for all x (b) It is periodic function
 (c) It is differential function for all x (d) It is neither an even nor an odd function
52. Let $f(x)$ be a non-constant twice differentiable function defined on $(-\infty, \infty)$ such that $f(x) = f(1 - x)$ and $f'\left(\frac{1}{4}\right) = 0$. Then, which of the following is (are) TRUE?
 (a) $f''(x)$ vanishes at least twice in $[0, 1]$ (b) $f'\left(\frac{1}{2}\right) = 0$
 (c) $\int_{-1/2}^{1/2} f\left(x + \frac{1}{2}\right) \sin x \, dx = 0$ (d) $\int_0^{1/2} f(t)^{\sin \pi t} \, dt = \int_{1/2}^1 f(1 - t) e^{\sin \pi t} \, dt$
53. If $I_n = \int_0^1 \frac{dx}{(1 + x^2)^n}$, where $n \in N$, then which of the following statements hold good?
 (a) $2nI_{n+1} = 2^{-n} + (2n - 1)I_n$ (b) $I_2 = \frac{\pi}{8} + \frac{1}{4}$
 (c) $I_2 = \frac{\pi}{8} - \frac{1}{4}$ (d) $I_3 = \frac{3\pi}{32} + \frac{1}{4}$
54. A function f is defined by $f(x) = \int_0^{3\pi} \cos z \cos(x - z) \, dz, 0 \leq x \leq 4\pi$. Then, which of the following hold(s) good?
 (a) Maximum value of f is $\frac{3\pi}{2}$
 (b) Minimum value of f is $-\frac{3\pi}{2}$
 (c) $f(x)$ is continuous but not differentiable in $(0, 4\pi)$
 (d) There exist at least one $c \in (0, 4\pi)$ such that $f'(c) =$
55. Let $f : (-1, 1) \rightarrow R$ be a continuous function such that $f(x) = x^2 e^{\frac{1}{1-x^2}}$. Then which of the following statement(s) is(are) TRUE?
 (a) The curve $y = f(x)$ is decreasing in $(-1, 0)$
 (b) The curve $f(x) = 1$ has two solution in $(-1, 1)$
 (c) The curve $y = f(x)$ is increasing in $(0, 1)$
 (d) The curve $f(x) = 1$ has no solution in $(-1, 1)$
56. Let $g(x) = x^a e^{ax}$ and $f(x) = \int_0^x t e^{2t} (1 + 3t^2)^{1/2} \, dt$. If $L = \lim_{x \rightarrow \infty} \frac{f'(x)}{g'(x)}$ is non-zero finite number. Then, which of the following is (are) TRUE?
 (a) The value of a is $\frac{3}{2}$ (b) The value of L is $\frac{\sqrt{3}}{2}$
 (c) The value of a is 2 (d) The value of L is $\frac{\sqrt{3}}{4}$

57. Let $f(x) = ax^2 + bx + c, a > 0$ and $f(2-x) = f(2+x), \forall x \in R$ and $f(x) = 0$ has 2 distinct real roots. Then, which of the following is(are) TRUE?
- (a) At least one root must be positive
 (b) $f(2) < f(1) < f(0)$
 (c) Minimum value of $f(x)$ is negative
 (d) vertex of graph of $y = f(x)$ lies in IIIrd quadrant
58. Which of the following is/are correct?
- (a) Between any two roots of the $e^x \cos x = 1$, there exists at least one root of $\tan x = 1$
 (b) Between any two roots of $e^x \sin x = 1$, there exists at least one root of $\tan x = -1$
 (c) Between any two roots of $e^x \cos x = 1$, there exists at least one root of $e^x \sin x = 1$
 (d) Between any two roots of $e^x \sin x = 1$, there exists at least one root of $e^x \cos x = 1$
59. $f(x) = \frac{[x]+1}{\{x\}+1}$ for $f: \left[0, \frac{5}{2}\right) \rightarrow \left(\frac{1}{2}, 3\right]$, where $[\cdot]$ and $\{\cdot\}$ represents part of x respectively. Then, which of the following is (are) TRUE?
- (a) $f(x)$ is injective discontinuous function
 (b) $f(x)$ is surjective non-differentiable function
 (c) $\min\left(\lim_{x \rightarrow 1^-} f(x), \lim_{x \rightarrow 1^+} f(x)\right) = f(1)$
 (d) $\max(x \text{ values of point of discontinuity}) = f(1)$
60. Let $\lambda = \int_{\pi/4}^{\pi/3} \frac{\sin x}{x} dx$. Then which of the following statements are true?
- (a) $\frac{\sqrt{3}}{8} \leq \lambda \leq \frac{\sqrt{2}}{6}$ (b) $\lambda < \frac{1}{4\sqrt{2}}$ (c) $\lambda > \frac{1}{2\sqrt{3}}$ (d) $\frac{1}{6} < \lambda < \frac{\pi}{12}$
61. Let $F: R \rightarrow R, g: R \rightarrow R$ and $h: R \rightarrow R$ be derivable functions such that $f(x) = x^3 + 4x + 3, g(f(x)) = x$ and $h(g(g(x))) = x$ for all $x \in R$, then
- (a) $h(0) = 42$ (b) $h(g(1)) = 8$ (c) $h'(0) = 124$ (d) $h'(1) = 1372$
62. Which of the following statement(s) is/are true?
- (a) If f is a continuous real valued function defined on $[0, 1]$ then $\exists$ positive number c such that $|f(x) - f(y)| \leq c$ for all x, y in $[0, 1]$
 (b) If f is continuous on R such that $\lim_{x \rightarrow -\infty} f(x) = -\infty$ and $\lim_{x \rightarrow \infty} f(x) = \infty$, then the equation $f(x) = k, k \in R$ has at least a root in R
 (c) If f and g are derivable and $f(x) > g(x)$ for $a < x < b$, then $f'(x) > g'(x)$ for $a < x < b$
 (d) if f and g are continuous and $f(x) > g(x)$ for $a < x < b$, then $\int_a^b f(x) dx > \int_a^b g(x) dx$
63. Let g be a continuous function such that $f(x) = \int_0^x g(y) dy$ and $h(x) = \int_0^x f(t) dt$. Then $\int_0^b x^2 f(x) dx =$
- (a) $\int_0^b 2x[f(b) - f(x)] dx$ (b) $\int_0^b 2x[h(b) - h(x)] dx$
 (c) $\int_0^b \left(\frac{b^3 - x^3}{3}\right) g(x) dx$ (d) $\int_0^b \left(\frac{b^3}{3} - x^2\right) g(x) dx$

64. Let $f(x) = \max\{1 + \sin x, 1, 1 - \cos x\}$, $x \in [0, 2\pi]$ and $g(x) = \max\{1, |x - 1|\}$, $x \in R$; then
 (a) $g(f(0)) = 1$ (b) $g(f(1)) = 1$ (c) $f(f(1)) = 1$ (d) $\Rightarrow a = -2$
65. Let $f(x) = \begin{cases} x+1, & 0 \leq x \leq 1 \\ 2x^2 - 6x + 6, & 1 < x \leq 2 \end{cases}$ and $g(t) = \int_{t-1}^1 f(x)dx$ for $t \in [1, 2]$
 Which of the following hold(s) good?
 (a) $f(x)$ is continuous and differentiable in $[0, 2]$
 (b) $g'(t)$ vanishes for $t = 3/2$ and 2
 (c) $g(t)$ is maximum at $t = 3/2$
 (d) $g(t)$ is minimum at $t = 1$
66. Let $\alpha(n)$ be the number of solution of the equation $\frac{d}{dx}(\cos^n x \cos nx) = 0$, where $n \in N$ and $n > 1$ in $[0, \pi]$. Which of the following statement(s) is (are) true?
 (a) $\alpha(n)$ is never even
 (b) $\alpha(2k) = \alpha(1k+1)$, $k \in n$
 (c) if $\alpha(n) - n = 2$, then n is even ($n \in N$)
 (d) If $\alpha(n) + n = \lambda$, then n can take two value ($\lambda, n \in N$)
67. If $f(x) = (a^2 - 2a + 1)e^x + (b^2 - 4b + 4)\ln x + (c^2 - 6c + 9)x^3 + 2\sin^{-1}x$ is a bounded function then
 (a) minimum value of $ax^2 + bx + c$ will be 2
 (b) $\tan^{-1}a + \tan^{-1}b + \tan^{-1}c = \pi$
 (c) number of solution of $f(x) = x$ will be 3
 (d) If graph of $y = g(x)$ is reflection of graph of $y = f(x)$ w. r. t. line $y=x$ then $g(\pi) = 1$
68. Let $f(x) = (x-1)^4(x-2)^3(x-3)^2(x-4)$ and $x^4z + xz^4 = (2x+3z)^5$ then $f'''(1)$ is equal to
 (a) $f''(2)$ (b) $f'(3)$ (c) $\left. \frac{d^2z}{dx^2} \right|_{x=0}$ (d) $\left. \frac{d^2z}{dx^2} \right|_{x=1}$
69. Let $f(x) = (g(x))^2 - a g(x) + 3 - a$.
 (a) If $g(x) = \ln x$ then complete range of values of a so that $f(x) \leq 0$ for at least one real $x \in (1, \infty)$ is $[2, \infty)$
 (b) If $g(x) = \ln x$ then complete range of values of a so that $f(x) \leq 0$ for at least one real $x \in (1, \infty)$ is $(-\infty, -6] \cup [2, \infty)$
 (c) If $g(x) = |x|$ then complete range of values of a such that $f(x) = 0$ has four distinct real roots is $(2, 3)$
 (d) If $g(x) = |x|$ then complete range of value of a such that $f(x) = 0$ has four distinct real root is $(3, \infty)$
70. If $x f^2(x) - (x^2 - x + 2)f(x) = x(x-2) \forall x \in R$ and $f(2) \neq 0$ then $\int_0^2 \left(\frac{f(x) + f(-x) + f''(x)}{2 - f(x)} \right) f(x) dx$ is equal to
 (a) $\int_{-1}^1 f(x) dx$ (b) $\int_0^2 f(x) dx$ (c) $\int_0^1 (f(x) + f(-x)) dx$ (d) $2 \int_0^1 f(x) dx$
71. If $f(x) = \begin{cases} \operatorname{sgn}(x^2 + kx + k), & x > 0 \\ a + bx - x^2, & x \leq 0 \end{cases}$ is continuous for all real x but derivable at exactly one point then
 (a) Number of integral values of k is less than or equal to 4
 (b) Number of integral values of k is more than 4 .
 (c) $a + b$ can be 1
 (d) $a + b$ can be 2

72. Let $f(x)$ and $g(x)$ be two polynomials on real numbers as $f(x) = ax^3 + bx^2 + cx + 1$ and $g(x) = px^3 + qx + r$. Given $a^2 + b^2 + c^2 + ab + bc + ca \leq 0$ and $g(0) = 2, g'(0) = -3, g''(0) = 2$ then
- (a) Value of $f(1) + g(1)$ is equal to 1
- (b) $\int_0^1 \frac{f(x)}{g(x) + 3x + 1} dx$ is equal to $\frac{\pi}{4}$
- (c) $\int_0^1 \frac{g(x)}{f(x)} dx$ is equal to $\frac{5}{6}$
- (d) Value of $f(2) + g(2)$ is equal to 1
73. If $\int (x^6 + 7x^5 + 6x^4 + 5x^3 + 4x^2 + 3x + 1)e^x dx = f(x)e^x + C$, where C is constant integration, $f(0) = 0$ and a continuous function $g(x)$ defined as $x(x+1)g(x) = f(x)$ then
- (a) $f(-1) = 1$ (b) $f(1) = 6$
- (c) $g(x)$ increases for $x > 0$ (d) $g(x)$ decreases for $x < 0$
74. Let $f : (0, \infty) \rightarrow (3, \infty]$ be a function satisfying the differentiable equation $xy' + (x+1)y = (3x^2 + 3x^3)e^2$. If $f(1) = e^2 + \frac{2}{e}$, then which of the following is/are correct?
- (a) $y = f(x)$ is onto function
- (b) $f(x) = \frac{2e^{-x}}{x} + 1$ has two distinct real roots
- (c) $f(x) = \frac{2e^{-x}}{x} + 1$ has one real roots
- (d) $f(x) = x^2 e^{2x} + 1$ has one real roots
75. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = (|x-1||x-2||x+1||x-3|)\sin \pi x$, then which of the following is/are correct
- (a) $y = f(x)$ is continuous but not differentiable
- (b) $y = f(x)$ is differentiable
- (c) $f''(x) = 0$ has at least 3 distinct real roots
- (d) $y = f(x)$ is onto function

PARAGRAPH TYPE

Passage-1

The tangent at point P of a curve $y = f(x)$ meets the y -axis at A , x -axis at B and the line parallel to y -axis through P meets x -axis at C such that area of ΔPBC is equal to the ratio of ordinate to abscissae of point P where $f(1) = 1$ and $f(3) = e^2$.

76. If the area bounded by the curve $y = f(x)$ line $x + y = 2$ and y -axis in the 1st quadrant is A , then
- (a) $\frac{1}{2} < A < 1$ (b) $1 < A < 2$ (c) $2 < A < 3$ (d) $3 < A < 4$
77. The value of $\int_{-2013}^{2013} \text{sgn}(f(x-2013)) dx$ is (where $\text{sgn}(\cdot)$ denotes signum function)
- (a) 0 (b) 2013 (c) 2026 (d) 4026

Passage-2

Let $z = f(x, y)$ be a continuous function of two independent variables x and y . In this function, if y remains unchanged and x alone varies then z becomes a continuous function of x alone. If this function possesses derivative, then this derivative is called partial derivative of z w.r.t. x . In other words, the partial differential coeff. of $z = f(x, y)$ w. r. t. x is defined as $\lim_{\delta x \rightarrow 0} \frac{f[(x + \delta x), y] - f(x, y)}{\delta x}$, provided

this limit exists is symbolically denoted by $\frac{\partial f}{\partial x}$ or $\frac{\partial z}{\partial x}$ or f_x .

Similarly, the partial differential coefficient of $z = f(x, y)$ w. r. t. y is defined as

$\lim_{\partial y \rightarrow 0} \frac{f[x, (y + \partial y)] - f(x, y)}{\partial y}$, provided this limit exists and written as $\frac{\partial f}{\partial y}$ or $\frac{\partial z}{\partial y}$.

f_x and f_y are called the first order partial differential coefficients of $z = f(x, y)$.

78. If $u = \log(\sec x + \sec y + \sec z)$, then $\sum \cot x \cdot \frac{\partial u}{\partial x}$ is equal to
(a) 0 (b) -1 (c) 1 (d) 1/2
79. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$
(a) $\frac{1}{x+y+z}$ (b) $\frac{3}{x+y+z}$ (c) $\frac{1}{x^3+y^3+z^3}$ (d) $\frac{1}{(x+y+z)^3}$

Passage-3

Let $f(x)$ is a continuous function satisfying $x^4 - 4x^2 \leq f(x) \leq 2x^2 - x^3 \forall x \in [0, 2]$ such that the area bounded by $y = f(x)$, $y = x^4 - 4x^2$, y -axis and the line $x = t$, where $0 \leq t \leq 2$ is λ time the area of the region bounded by $y = f(x)$, $y = 2x^2 - x^3$, y -axis and the line $x = t$, where $0 \leq t \leq 2$.

80. The function $f(x)$ is given by
(a) $\frac{1}{\lambda+1} [x^4 - \lambda x^3 + 2(\lambda-2)x]$ (b) $\frac{1}{\lambda+1} [x^4 - \lambda x^3 + 2(\lambda-2)x^2]$
(c) $\frac{1}{\lambda+1} [x^4 - (\lambda-2)x^3 + 2(\lambda-2)x^2]$ (d) $\frac{1}{\lambda+1} [x^4 - (\lambda-2)x^3 + 2(\lambda-2)x]$
81. The value of $\int_{-1}^1 f(x) dx$ is
(a) $\frac{2}{15(\lambda+1)} (13 + 11\lambda)$ (b) $\frac{2}{15(\lambda+1)} (10\lambda - 17)$
(c) $\frac{2}{15(\lambda+1)} (11 - 17\lambda)$ (d) none of these

Passage-4

The method of differentiation under integral sign can be used to evaluate those integral that are not easily integrable otherwise.

Its application falls into two classes

Class I: Given integral can be obtained by reduction by differentiating some other integral which can be easily arrived at.

Class II: Given integral can be found by first differentiating the integral, then evaluating new integral thus obtained and eventually integrating the result.

82. The value of $\int_0^{\pi/2} \log(\alpha^2 \cos^2 \theta + \beta^2 \sin^2 \theta) d\theta$, is equal to
- (a) $\frac{\pi}{2} \ln(\alpha + \beta)$ (b) $\pi \ln\left(\frac{\alpha + \beta}{2}\right)$ (c) $2\pi \ln\left(\frac{\alpha + \beta}{2}\right)$ (d) $2\pi \ln(\alpha + \beta)$
83. The value of $\int_0^{\pi/2} \frac{dx}{(a^2 \sin^2 x + b^2 \cos^2 x)^2}$ is
- (a) $\frac{\pi(a^2 + b^2)}{2a^3b^3}$ (b) $\frac{\pi(a^2 + b^2)}{3a^2b^2}$ (c) $\frac{\pi(a^2 + b^2)}{2a^2b^2}$ (d) none of these

Passage-5

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = e^x - e^{-2x} + 3x + 2$ and g be the inverse function of f .

84. Identify which of the following statement(s) is (are) correct?
- (a) If $f(\alpha) = 0$, then $[\alpha] = 1$ (b) $f(\alpha) = 0$, then $[\alpha] = -1$
- (c) $\lim_{x \rightarrow \infty} \frac{f(g(x))}{f(x)} = 1$ (d) $\lim_{x \rightarrow \infty} \frac{f(g(x))}{f(x)} = \frac{1}{4}$
- [Note: $[k]$ denotes greatest integer function less than or equal to k .]
85. If $h(x) = g(g(x))$ then
- (a) $h'(2) = \frac{g'(0)}{6}$ (b) $h'(2) = 6g(0)$
- (c) $h'(e^2 - e^{-4} + 8) = \frac{1}{6(e^2 + 2e^{-4} + 3)}$ (d) $h'(e^2 - e^{-4} + 8) = \frac{1}{6(e^2 + e^{-4} + 3)}$

Passage-6

Let f, g, h be three functions such that $f'(x) > 0$ and $g(x) \geq 0, \forall x \in \mathbb{R}$. Where $\int f(x)g(x)dx = \frac{x^4}{4} + C_1$

and $\int \frac{f(x)}{g(x)} dx = \int \frac{g(x)}{h(x)} dx = \ln|x| + C_2$ [where C_1, C_2 are constants of integration and $g(x) \cdot h(x) \neq 0$].

86. If $I = \int_0^2 f(x)g(x)h(x)dx$, then $[I]$ is equal to
- (a) 3 (b) 9 (c) 18 (d) 36
87. If $\sin^{-1}f(x) + \cos^{-1}g(x) + \frac{\pi}{2} = 0$ has a solution for $x = \alpha$ then $\int_0^\alpha h(x)dx$
- (a) $\frac{1}{4}$ (b) $-\frac{1}{4}$ (c) $\frac{1}{3}$ (d) 0

Passage-7

Let $y = f(x)$ be a polynomial function of degree 4, which is symmetrical about the line $x=1$ and touches the x -axis and straight line $y+2=0$ and distance between both absolute minima is 4

88. The value of $f(0) + f(1) + f(2)$ is equal to
- (a) -7 (b) $-\frac{7}{4}$ (c) $-\frac{7}{8}$ (d) $-\frac{9}{4}$

89. The value of $\int \frac{dx}{f(x+1)}$ is equal to

(a) $\frac{1}{2\sqrt{2}} \ln \left| \frac{x+2\sqrt{2}}{x-2\sqrt{2}} \right| - \frac{1}{x} + k$

(b) $\frac{1}{2\sqrt{2}} \ln \left| \frac{x+2\sqrt{2}}{x-2\sqrt{2}} \right| + \frac{1}{x} + k$

(c) $\frac{1}{2\sqrt{2}} \tan^{-1} \left(\frac{x}{2\sqrt{2}} \right) + \frac{1}{x} + k$

(d) $\frac{1}{2\sqrt{2}} \tan^{-1} \left(\frac{x}{2\sqrt{2}} \right) - \frac{1}{x} + k$

(Note: where k is the constant of integration.)

Passage-8

Let $f(x) = ax^3 + bx^2 + cx + d$, where a, b, c, d are non-zero real numbers and $g(x) = \int_0^x (1 + e^t) f(t) dt$,

where $g(1) = g(2) = g(3) = 0$ then

90. Sum of all the real roots of $f(x) = 0$ lies in the interval

- (a) (3, 6) (b) (5, 8) (c) (6, 12) (d) (-3, 3)

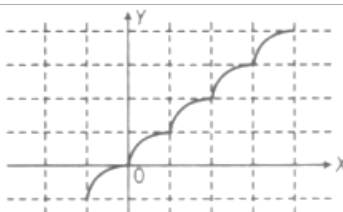
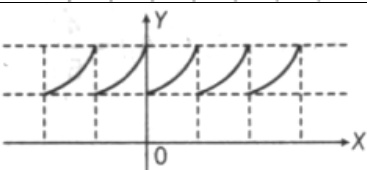
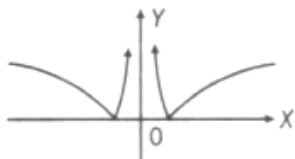
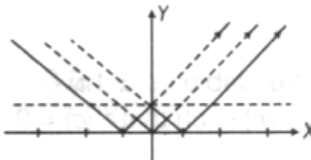
91. If α be point of maximum and β be point of minimum of $y = f(x)$ then $\alpha\beta$ lies in the interval

- (a) (-2, 0) (b) (0, 6) (c) (2, 4) (d) (4, 6)

Passage-9

By appropriately matching the information given the three columns of the following table.

Column I, II and III contains functions their graph and nature respectively

Column I	Column II	Column III
A. $f(x) = \left \frac{1}{x} \right - 3$	I. 	P. One-one
B. $f(x) = [x] + \sqrt{x - [x]}$	II. 	Q. Onto
C. $f(x) = \min \{ x , x-1 , x+1 \}$	III. 	R. Into
D. $f(x) = \frac{2^x}{2^{\{x\}}}$	IV. 	S. Many-one

92. When $x \in [0, 1]$ and $f(x) \in [0, 1]$, then which of the following is correctly matched?

- (a) B-I-P (b) A-III-P (c) C-IV-P (d) D-II-P

93. When $x \in [-1, 1]$ and $f(x) \in [0, 2]$, then which of the following is correctly matched?

- (a) A-III-S (b) B-I-P (c) C-IV-S (d) D-II-P

Passage-10

Suppose a function $f(x)$ satisfies the following conditions $f(x+y) = \frac{f(x)+f(y)}{1+f(x).f(y)}, \forall x, y$ and $-1 < f(x) < 1, \forall x \in R$. Then,

94. $f(x)$ is differentiable over the set
 (a) R (b) $R - \{-1, 0, 1\}$ only (c) $\{-1, 0, 1\}$ only (d) None of these
95. The value of the limit $\lim_{x \rightarrow \infty} [f(x)]^x$ is
 (a) 0 (b) 1 (c) -1 (d) 2

MATRIX TYPE

Matrix-1

96. Consider $f(x) = x^2 - x + k - 2, k \in R$ and $g(x) = x^2 - x + 1$

Column-I		Column-II	
(A)	If the complete set of values of k for which $y = f(x) $ is non derivable at 5 distinct points is (a, b), then $8(b-a) =$	(P.)	6
(B)	If $k \neq 3$, then number of tangents drawn to the curve $y = \frac{f(x)}{g(x)}$ which are parallel to x-axis is	(Q.)	1
(C)	If the complete set of values of k for which $\left[\frac{f(x)}{g(x)} \right]$ (where $[.]$ denotes greatest integer function) is continuous $\forall x \in R$ is $[\ell, m]$, then $4(m-\ell) =$	(R.)	2
(D)	If $ f(x) + g(x) = f(x) + g(x) $ is true for all real values of x, then smallest integral value of k is	(S.)	3

Matrix-2

97. Match the following

Column-I		Column-II	
(A)	Area bounded by $f(x) = \frac{(x-1)(x+1)}{x-2}$, then x-axis and ordinates $x=0$ and $x = \frac{3}{2}$	(P.)	$-\frac{3}{4}$
(B)	If $y = f(x)$ makes positive intercepts of 2 and 1 units on coordinate axes respectively and encloses an area of $\frac{3}{4}$ sq. units with the axes, then the value of $\int_0^2 xf'(x)dx$ is equal to	(Q.)	$\frac{4}{3}$
(C)	The area of the region bounded by the parabolas $x + 2y^2 = 0$ and $3y^2 + x + 2 = 2$ is equal to	(R.)	$\frac{1}{4}$
(D)	If the ordinate $x = 2\sqrt{2}$ divides the area bounded by the X-axis, the curve $y = 1 + \frac{8}{x^2}$ and the ordinates $x = 2, x = 4$ into two parts whose area are equal to A_1 and A_2 , then value of $\frac{1}{A_1 A_2}$ is equal to	(S.)	$\frac{7}{8}$

Matrix-3

98. Consider $f_1(x) = \cos^{-1}\left(\frac{x^2-1}{1+x^2}\right)$, $f_2(x) = 3 \tan^{-1}x$, $f_3(x) = f_1(x) - |f_2(x)|$ and $f_4(x) = f_1(x) - f_2(x)$.

Column-I		Column-II	
(A)	$\left. \frac{d(f_2(x))}{d(f_1(x))} \right _{x=-\frac{3\pi}{4}}$ is equal to	(P.)	1
(B)	$\left. \frac{d(f_4(x))}{d(f_2(x))} \right _{x=2}$ is equal to	(Q.)	$-\frac{5}{3}$
(C)	$\left. \frac{d(f_3(x))}{d(f_1(x))} \right _{x=-2}$ is equal to	(R.)	$\frac{3}{2}$
(D)	$\left. \frac{d(f_3(x))}{d(f_4(x))} \right _{x=\pi}$ is equal to	(S.)	$\frac{5}{2}$

Matrix-4

99. Consider $f(x) = \left| -|x|^2 + 2|x| \right|$ and $g(x) = \begin{cases} \min.\{f(t)\}, & -3 \leq t \leq x, & -3 \leq x < -1 \\ \min.\{f(t)\}, & -1 \leq t \leq x, & -1 \leq x < 0 \\ \min.\{f(t)\}, & 0 \leq t \leq x, & 0 \leq x < 1 \\ \max.\{f(t)\}, & 1 \leq t \leq x, & 1 \leq x \leq 3 \end{cases}$, then

Column-I		Column-II	
(A)	The value of $g'(-2)$ is	(P.)	3
(B)	The value of $g'(1)$ is	(Q.)	5
(C)	The number of point where g is discontinuous in $[-3, 3]$ is	(R.)	6
(D)	The number of point where g is non-derivable in $(-3, 3)$ is	(S.)	DNE

Matrix-5

100. Match the following :

Column-I		Column-II	
(A)	If $\int_0^1 f(x)dx = 1$ and $f(2x) = 2f(x)$, then $3 \int_1^2 f(x)dx$ is equal to	(P.)	11
(B)	If $\int_0^2 f(x)dx = 2$ and $f(2x) = 3f(x)$, then $\int_2^4 f(x)dx$ is equal to	(Q.)	10
(C)	If $\int_2^1 f(x)dx = 1$ and $f(3x) = 3f(x)$, then $\int_1^3 f(x)dx$ is equal to	(R.)	9
(D)	If $\int_0^1 f(x)dx = 1$ and $f(3x) = 4f(x)$, then $\int_1^3 f(x)dx$ is equal to	(S.)	8

NUMERICAL GRID

101. Let $\int \frac{f'(x)g(x) - g'(x)f(x)}{(f(x) + g(x))\sqrt{f(x)g(x) - g^2(x)}} dx = \sqrt{m} \tan^{-1} \left(\sqrt{\frac{f(x) - g(x)}{ng(x)}} \right) + C$
Where $m \in N$ and 'C' is constant of integration ($g(x) > 0$). Find the value of $(m^2 + n^2)$
102. Let S denote sum of the series $\frac{3}{2^3} + \frac{4}{2^4 \cdot 3} + \frac{5}{2^6 \cdot 3} + \frac{6}{2^7 \cdot 5} + \dots \infty$. Then compute the value of S^{-1} .
103. Let f be a polynomial function satisfying $2 + f(x)f(y) = f(x) + f(y) + f(xy) \forall x, y \in R$.
If $f(2) = 5$, then find the value of $10[f(f(2)) + 1]$.
104. Let $f: R \rightarrow R$ be a function satisfying
 $f(x + 2y) = f(x)e^{2y} + f(2y)e^x + x^2(1 - e^{2y}) + 4y^2(1 - e^x) + 4xy \forall x, y \in R$.
If $f'(0) = 1$, then find the value of $\{99[f(1) - e]\}^2$.
105. If a polynomial $g(x)$ satisfies $xg(x-1) = (x-3)g(x)$ for all x and $g(3) = 6$, then find the value of $g(20)$.
106. Let $f(x) = x^4 + ax^3 + bx^2 + cx + d$, where a, b, c and d are constants. If $f(1) = 10, f(2) = 20, f(3) = 30$
then find the value of $\frac{9[f(10) + f(-6)]}{8}$.
107. Let $f(x) = \begin{cases} x+a, & \text{if } x < 0 \\ |x+1|, & \text{if } x \geq 0 \end{cases}$ and $g(x) = \begin{cases} x+1, & \text{if } x < 0 \\ (x+1)^2 + b, & \text{if } x > 0 \end{cases}$, where a and b are non-negative real numbers. If $(g \circ f)(x)$ is continuous for all x, then find the value of $1332(a+b)$
108. Let $f(x) = \begin{cases} a + |x-b|^2, & |x-b| \leq k \\ c + |x-b|, & |x-b| > k \end{cases}$ ($b > 0$) be a differentiable function for all x, then find the value of $10[4(a-c) + 8]^3$.
109. Let $f(x, y)$ be a periodic function, satisfying the condition $f(x, y) = f(2x + 2y, 2y - 2x) \forall x, y \in R$ and let $g(x)$ be a function defined as $g(x) = f(2^x, 0)$. If $g(x)$ is periodic function with periodic T, then find the value of $5T^3$.
110. Let S_1, S_2 be two circles of unit radius touching each other externally and L be one of the direct common tangent. A circle C_1 with radius r_1 is such that it touches the circles S_1, S_2 and the line L. Similarly, circles $C_2, C_3, \dots, C_n$ are the circle of radii $r_2, r_3, \dots, r_n$ where circle C_r touches $S_1 C_{r-1}$ and one of the direct common tangents between them.
If $\lim_{n \rightarrow \infty} \left(\frac{\frac{1}{r_1} + \frac{1}{r_2} + \dots + \frac{1}{r_n}}{n^2} \right) = \lambda$, then find the value of $80(3\lambda + 80)$.
111. Let $f\left(\frac{x_1 + x_2 + \dots + x_n}{n}\right) = \frac{f(x_1) + f(x_2) + \dots + f(x_n)}{n}$, where $x_i \in R (i = 1, 2, \dots, n)$ and $n \in N$. If $f(x)$ is differentiable and $f'(0) = a, f(0) = b$, then find the value of $100[f''(x) + 9]^2$.

112. If a_1, a_2 and a_3 are the three value of a which satisfy the equation $\int_0^{\pi/2} (\sin x + a \cos x)^3 dx - \frac{4a}{\pi-2} \int_0^{\pi/2} x \cos(x) dx = 2$, then find the value of $84(a_1^2 + a_2^2 + a_3^2)$.
113. Let P be a point on the parabola $y^2 = 4x$ with the ordinate y satisfying $1 < y \leq 2$. Then normal to the parabola at P intersects the x-axis in M. Let D be the maximum difference in the area of triangles PMN and PMS, where S is the focus of the parabola, then the value of $10(D^3 + 1)$.
114. Let I denotes the greatest integral value of the area of the region enclosed by the curve $\left[y - e^{\sqrt{1-x^2}} \right]^2 = 1 - x^2$. Then find the value of $(1000)I^2$
115. The area between the two curves by $y = |x - (x)|$ and $y = k \sin \pi x, k \geq 1$, in the interval $0 \leq x < 1$ is found to be equal to 2 sq. units. Fiend the value of $(k^2) + 2$, where $(.)$ denotes the integer nearest to x . For e. g.: $(1.3) = 1, (1.6) = 2$ and so on.
116. If I be the greatest value of the definite integral $\int_0^1 \sqrt{(1+x^2)(1+x^4)} dx$, then find the value of $(225)I^4$
117. Let $I = \int_1^{16} \tan^{-1} \sqrt{\sqrt{x} - 1} dx$, then find the value of $9(7[I] + 8)$, where $[.]$ denotes the greatest integer function.
118. Let $f(x) = ||x - 2| - 1| - 1$ and $g(x) = \begin{cases} \cos^{-1}(\text{sgn}(-x)); & x < 0 \\ \tan^{-1}(1 + \text{sgn}(x)); & x = 0 \\ \sin^{-1}(\text{sgn}|x|); & x > 0 \end{cases}$.
- If the sum of the elements in the range of $(g \circ f)(x)$ is $\left(\frac{p}{q}\right)\pi$,
Where $p, q \in \mathbb{N}$, then find the least value of $(p + q)$.
[Note: $\text{sgn}(y)$ denotes the signum function of y .]
119. If $f(x) = \begin{cases} -x; & x < 0 \\ |x - 2|; & x \geq 0 \end{cases}$, then find the number of solutions of the equation $f(f(x)) = 1$.
120. Let $f(x)$ be a monic polynomial function of degree 6 such that $f(1) = f(-1) = 5, f(2) = f(-2) = 8, f(3) = f(-3) = 13$, then find the value of $(f(\sqrt{2}) - f(\sqrt{3}))$.
121. If $g(x) = e^{f(x)}$ and $f(x+1) = x + f(x), \forall x \in \mathbb{R}$ are derivable function then find the value of $\frac{g'\left(\frac{7}{2}\right)}{g\left(\frac{7}{2}\right)} \cdot \frac{g'\left(\frac{1}{2}\right)}{g\left(\frac{1}{2}\right)}$.
122. Regular polygons are inscribed and circumscribed to a given circle of radius R:
Let $F(n)$ be area of inscribed $2n$ -sided regular polygon.
 $G(n)$ be area of circumscribed $2n$ -sided regular polygon
 $f(n)$ be area of inscribed n -sided regular polygon
 $g(n)$ be area circumscribed n -sided regular polygon.
- Then the value of $\lim_{n \rightarrow \infty} \left(\frac{g(n) - f(n)}{G(n) - F(n)} \right)$ is

123. Let $\lim_{h \rightarrow 0} \frac{h^2}{f(x+2h) - 2f(x+h) + f(x)} = \frac{x^{1-x}}{1+x(1+\ln x)^2}$. If $\lim_{x \rightarrow 0^+} f(x) = 1$ and $f(1) = 2$, then find the value of $\frac{f(3)}{f(2)}$.
124. Find the number of integral value of x for which the function $f(x) = [x^4] - [x]^4$ is continuous.
[Note: $[k]$ denotes greatest integer function less than or equal to k .]
125. Let $y = f(x)$ be a continuous function such that $\lim_{x \rightarrow \infty} \frac{\int_0^x f(t) dt}{x^2} = 1$ then find the value of $\lim_{x \rightarrow \infty} \frac{\int_0^x f(3t^2) dt}{x \int_0^x f(t) dt}$.

ANSWER KEY

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
b	b	d	a	d	a	d	b	a	d	c	b	a	c	b
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
a	a	b	c	b	a	d	d	d	a	a	d	d	a	c
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
b	c	d	b	b	a	d	d	a	d	a	c	b	b	c
46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
a	a	d	b	d	abc	abcd	abd	abd	abc	bc	abc	abc	abd	ad
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
abcd	abd	bc	abd	bcd	ab	abd	abcd	ac	ac	bd	acd	bcd	acd	bcd
76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
a	d	c	b	b	b	b	d	b	ac	c	a	b	b	a
91	92	93	94	95	96		97		98		99		100	
b	a	c	a	b	R,Q,P,S		S,P,Q,R		R,Q,S,P		S,S,P,Q		R,Q,S,P	
101	102	103	104	105	106	107	108	109	110	111	112	113	114	115
8	2	270	9801	6840	9117	1332	7290	8640	6480	8100	441	90	9000	18
116	117	118	119	120	121	122	123	124	125					
576	891	7	5	1	3	4	5	1	2					